Optimal Control and Optimization Methods for Multi-Robot Systems

Javier Alonso-Mora, Ketan Savla and Daniela Rus

Tutorial on Multi-robot systems @ RSS 2015

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Massachusetts Institute of Technology

USC University of Southern California

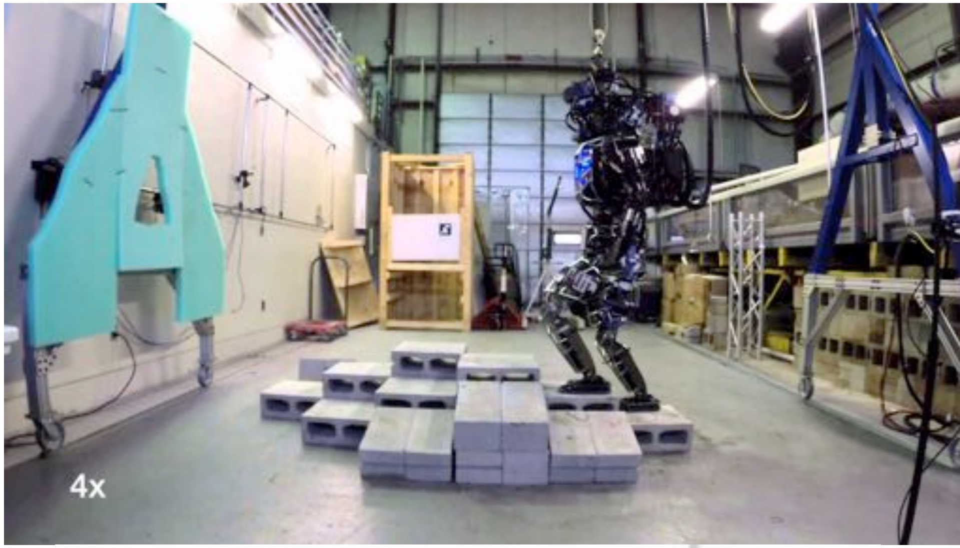
Future: many robots performing many tasks

We aim at optimal solutions for multi-robots

- Optimal control and optimization methods

- Attractive since:
  - they provide guarantees in the optimality of the solution
  - applicable to efficiently solve a wide range of problems
    - thanks to advances in the field of constrained optimization
    - and an increase in computational power of robotic platforms

Optimization is everywhere



Overview of this talk

- We give an overview of the required tools
- We focus on four canonical problems for multi-robot systems
- We describe some of the works by the community
- Disclaimers
  - Focus on motion planning / control / task assignment
  - Broad field – we will miss some things
  - Large body of works – if you feel we are missing some important reference, please let us know, We'll gladly add them
  - Contact: jalonsom@mit.edu
  - We are working on a tutorial/review

Overview

- Introduction
- **1. Optimal control and optimization tools**
  - **Optimal control & dynamic programming**
  - **Constrained optimization**
  - **Combinatorial optimization**
- 2. Problem definition & overview of state of the art
- Summary

Optimal control & dynamic programming

- Given a controlled dynamical system
  - State x(t), control input u(t)
  - Continuous

$$\dot{x} = f(x, u), \; x(0) = x^0$$

  - Discrete

$$x(t+1) = Ax(t) + Bu(t)$$

- A running cost $r(x(t), u(t))$

- Find the optimal control inputs

Optimal control & dynamic programming

- Optimal control [discrete, infinite horizon]

$$
\begin{aligned}
\text{minimize} \quad & J = \sum_{t=0}^{\infty} r(x(t), u(t)) \\
\text{subject to} \quad & u(t) \in \mathcal{U},\ x(t) \in \mathcal{X}, \quad t = 0, 1, \ldots \\
& x(t+1) = Ax(t) + Bu(t), \quad t = 0, 1, \ldots \\
& x(0) = x^0
\end{aligned}
$$

Running cost

State and control constraints

Controlled dynamical model

Initial state

- Dynamic programming solves for a value function satisfying Bellman equation

Model predictive control

- Model predictive control

$$
\begin{aligned}
\text{minimize} \quad & \sum_{\tau=t}^{t+T} r(x(\tau), u(\tau)) \\
\text{subject to} \quad & u(\tau) \in \mathcal{U},\ x(\tau) \in \mathcal{X},\ \ \tau = t, \ldots, t+T \\
& x(\tau+1) = Ax(\tau) + Bu(\tau),\ \ \tau = t, \ldots, t+T \\
& x(0) = x^0
\end{aligned}
$$

- Solve for a time horizon T and apply the first command, repeat at t+1

- Can be solved implicitly or explicitly (regions)

Constrained optimization

- For a set of variables

$$\mathbf{x} \in \mathbb{X}$$

- Find the optimal value that minimizes

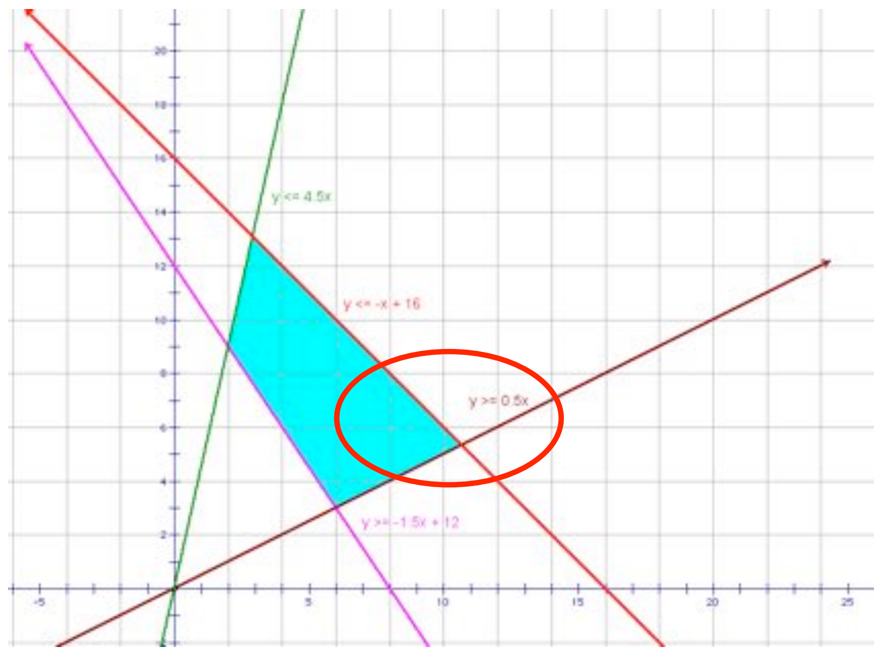
$$\begin{aligned} \mathbf{x}^* := \quad & \arg\min_{\mathbf{x}} \quad f(\mathbf{x}) \\ & \text{subject to} \quad g_i(\mathbf{x}) \leq 0 \quad \forall i \in \{1, \ldots, n_{ineq}\} \\ & \qquad\qquad\quad h_i(\mathbf{x}) = 0 \quad \forall i \in \{1, \ldots, n_{eq}\} \end{aligned}$$

Depending on the “shape” of f(x), g_i(x) and h_i(x) different problems are formulated

Constrained optimization

- ***Convex** optimization with **continuous** variables* $\boldsymbol{x} \in \mathbb{R}^{\nu}$
  - *Linear programming LP* $w_n x_1 + .. + w_n x_n$
  - *Quadratic programming QP* $w_n x_1^2 + .. + w_n x_n^2$
  - *Semi-definite programming SDP*

- convex optimization methods are (roughly) always global, always fast



Constrained optimization

- for general nonconvex problems
  - **local optimization methods** are fast, but need not find global solution (and even when they do, cannot certify it)
  - **global optimization methods** find global solution (and certify it), but are not always fast (indeed, are often slow)

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Constrained optimization

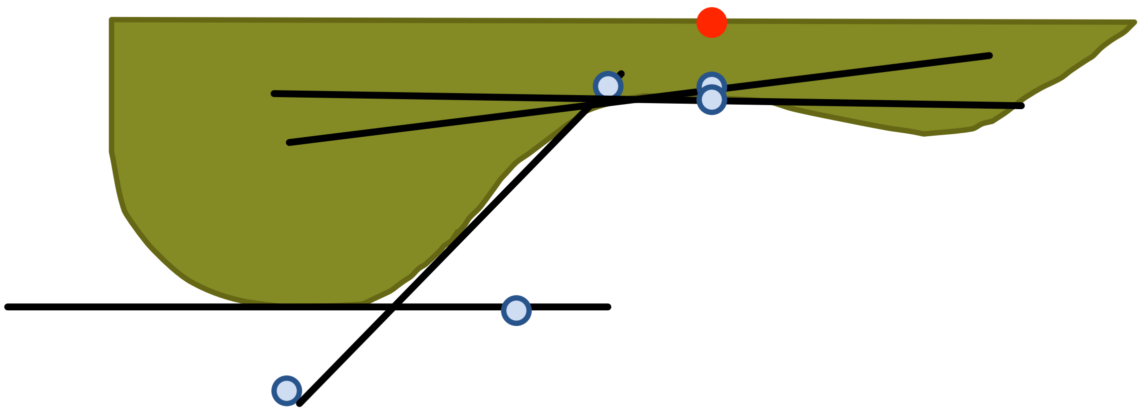
- ***Non-convex*** *optimization with* ***continuous*** *variables* $\boldsymbol{x} \in \mathbb{R}^{\nu}$
  - *Search techniques [global]*
  - *Gradient-based methods [local]*
  - *Sequential convex programming* ***SCP*** *[local]*

Constrained optimization

- ***Non-convex*** *optimization with* ***continuous*** *variables* $\boldsymbol{x} \in \mathbb{R}^{\nu}$
  - *Sequential convex programming* ***SCP*** *[local]*

EFFICIENT LOCAL OPTIMUM

- a local optimization method for nonconvex problems that leverages convex optimization
  - convex portions of a problem are handled 'exactly' and efficiently



Constrained optimization

- ***Non-convex*** *optimization with* ***continuous*** *variables* $\boldsymbol{x} \in \mathbb{R}^{\nu}$
  - *Sequential convex programming* ***SCP*** *[local]*

EFFICIENT LOCAL OPTIMUM

- a local optimization method for nonconvex problems that leverages convex optimization
  - convex portions of a problem are handled 'exactly' and efficiently
- SCP is a **heuristic**
  - it can fail to find optimal (or even feasible) point
  - results can (and often do) depend on starting point (can run algorithm from many initial points and take best result)
- SCP often works well, *i.e.*, finds a feasible point with good, if not optimal, objective value

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Constrained optimization

- *Optimization with **integer** variables*
  - *Integer linear program as network flow*
  - *Mixed integer program MIP [global]*

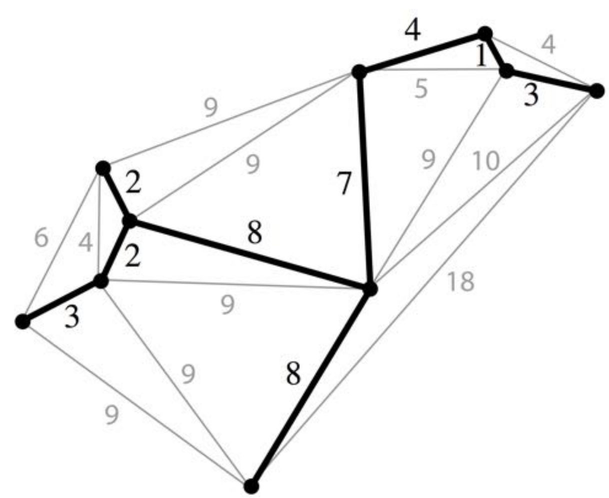
$x_j \in \mathbb{N}, \quad x_j \in \{0,1\}$

EFFICIENT - GLOBAL OPTIMUM
INEFFICIENT - GLOBAL OPTIMUM

- *Combinatorial optimization*
  - *Traveling salesman problem TSP*
    - *small problems solved via MIP, large problems solved with heuristics*

Branch-and-Bound

Each node in branch-and-bound is a new MIP



Constrained optimization: overview

- *Convex optimization with continuous variables*
  - *LP / QP / SDP*

VERY EFFICIENT GLOBAL OPTIMUM

- *Non-convex optimization with continuous variables*
  - *Gradient-based methods [local]*
  - *Sequential convex programming SCP [local]*

EFFICIENT LOCAL OPTIMUM

- *Optimization with integer variables*
  - *Mixed integer program MIP [global]*
  - *Integer linear program as network flow*
  - *Combinatorial optimization*

INEFFICIENT - GLOBAL OPTIMUM

EFFICIENT - GLOBAL OPTIMUM

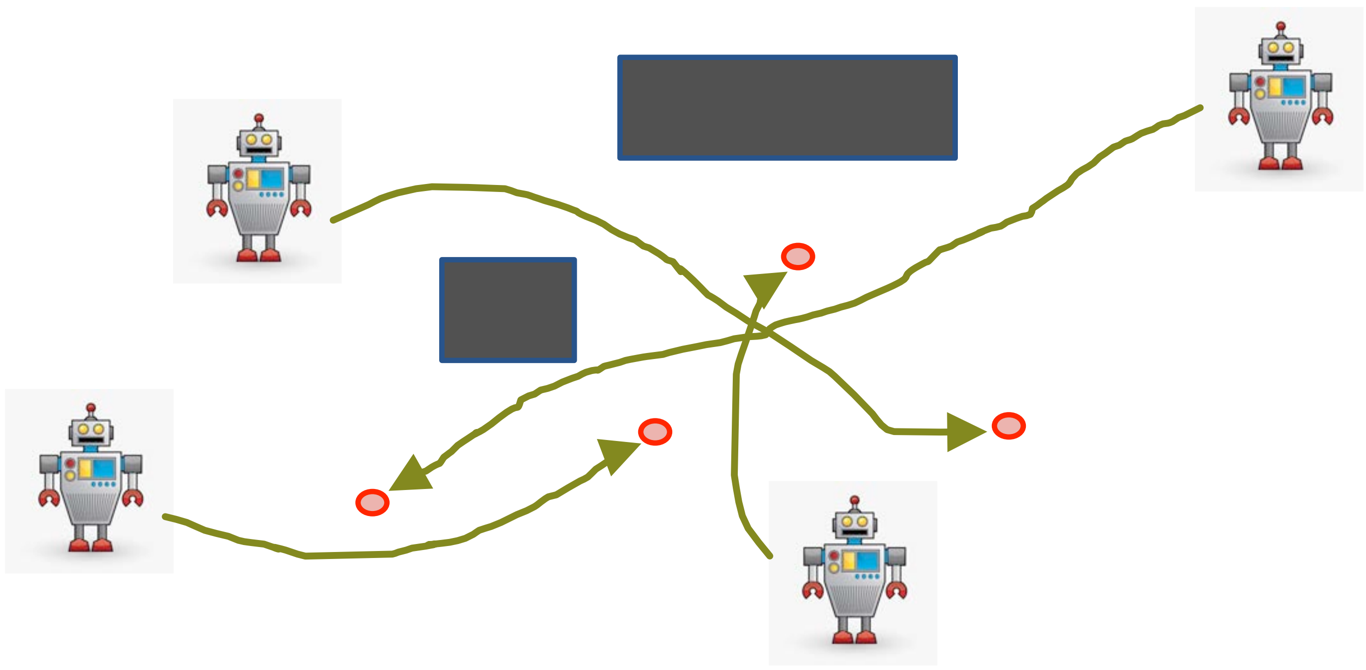
INEFFICIENT – TYPICALLY HEURISTIC

Overview

- Introduction
- 1. Optimal control and optimization tools
- **2. Problem definition & state of the art**
  - **Multi-robot motion planning**
  - **Formation planning**
  - **Task assignment**
  - **Surveillance and monitoring**
- Summary

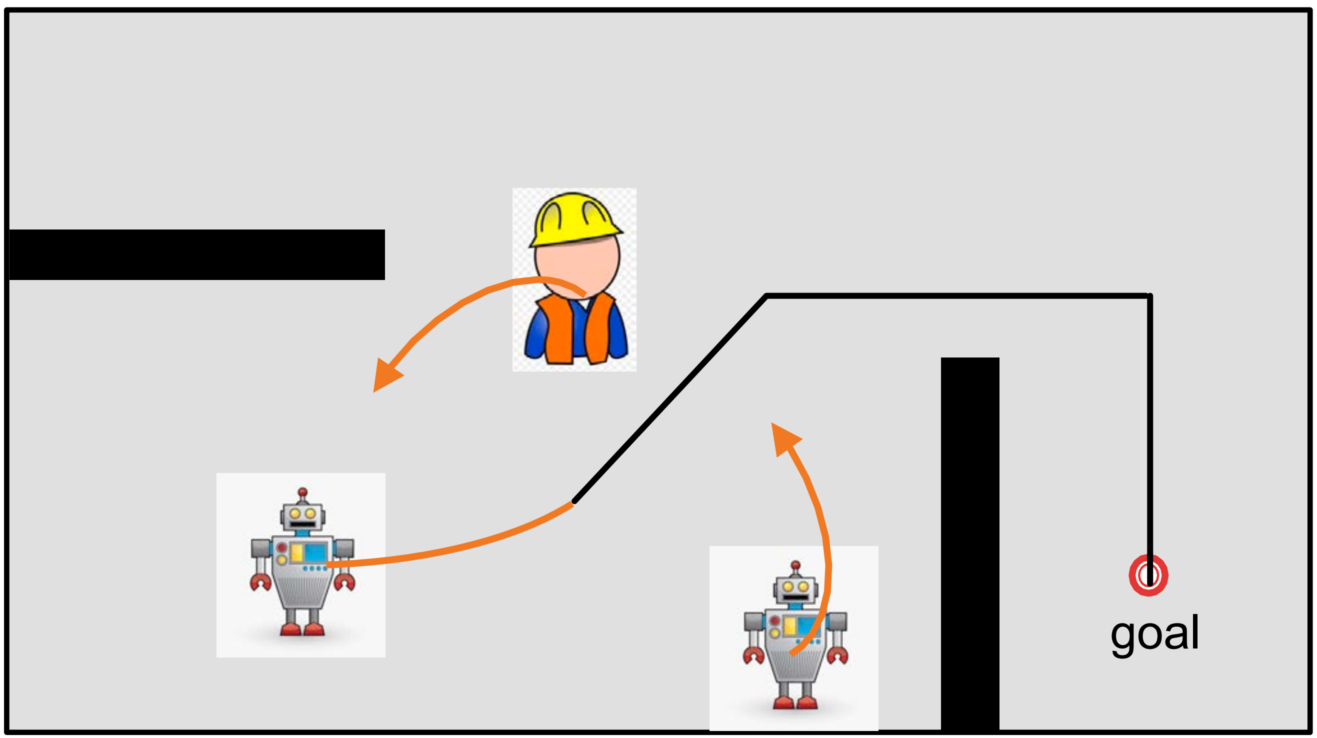
Multi-robot motion planning: problem definition

- Compute robot trajectories such that
  - Drive robots initial to final configuration
  - Avoid static and dynamic obstacles
  - Avoid inter-robot collisions
  - Respect dynamic model of the robot
    - Kinematic model, velocity/acceleration limits….



Multi-robot motion planning: problem definition

- Global planning
  - Trajectory from initial to final state
- Local planning (collision avoidance)
  - Trajectory from initial state up to a short time horizon



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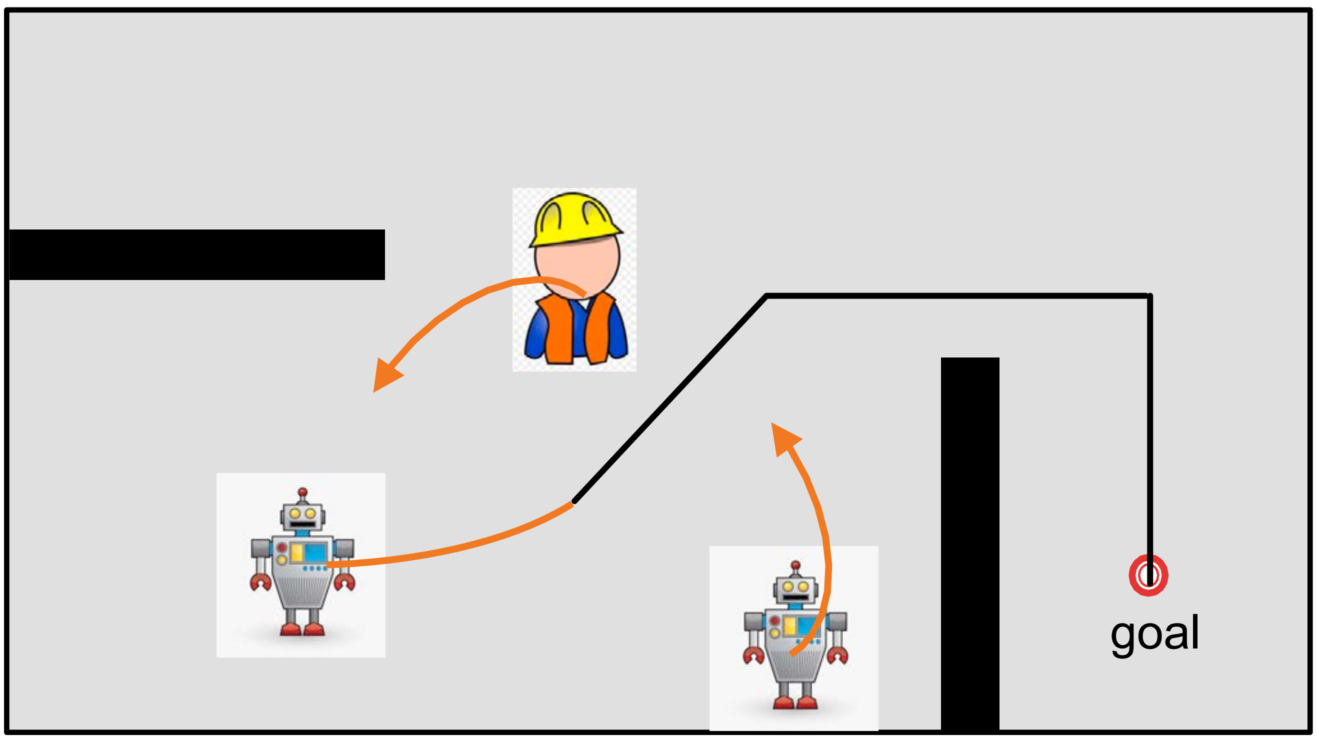
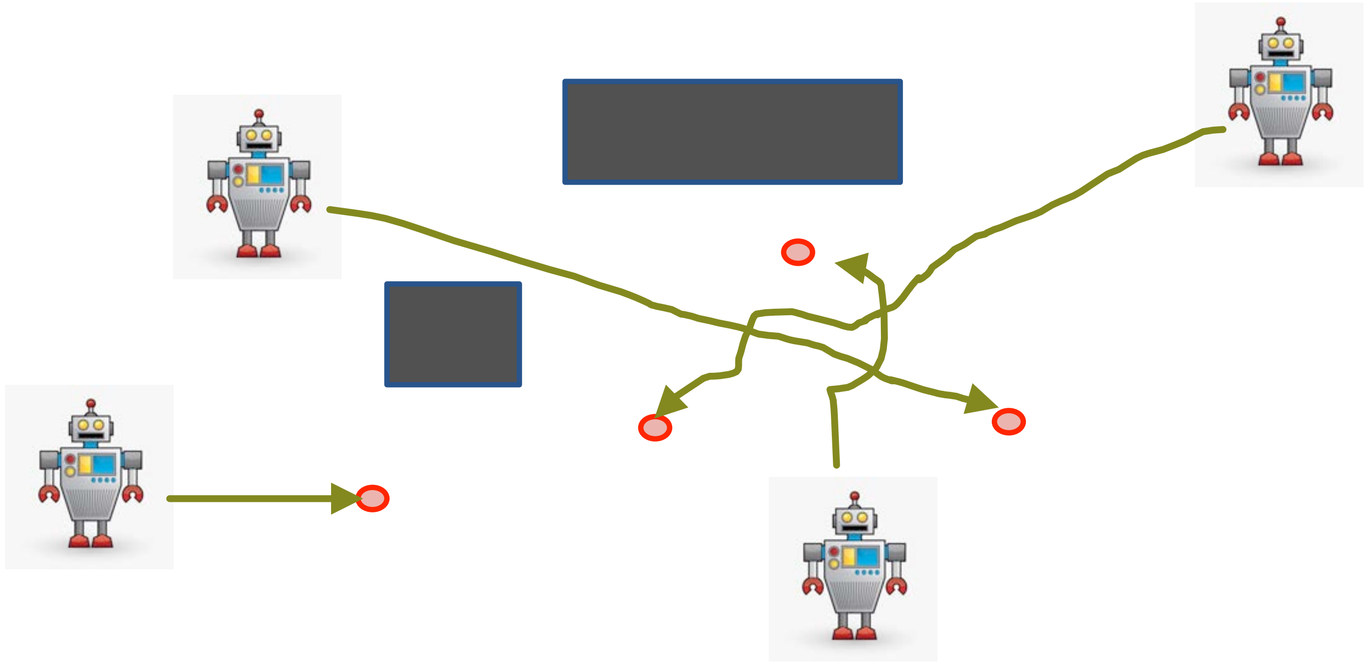


MMP: global planning

- "Traditional" approaches
  - Assign priorities and sequentially compute trajectories



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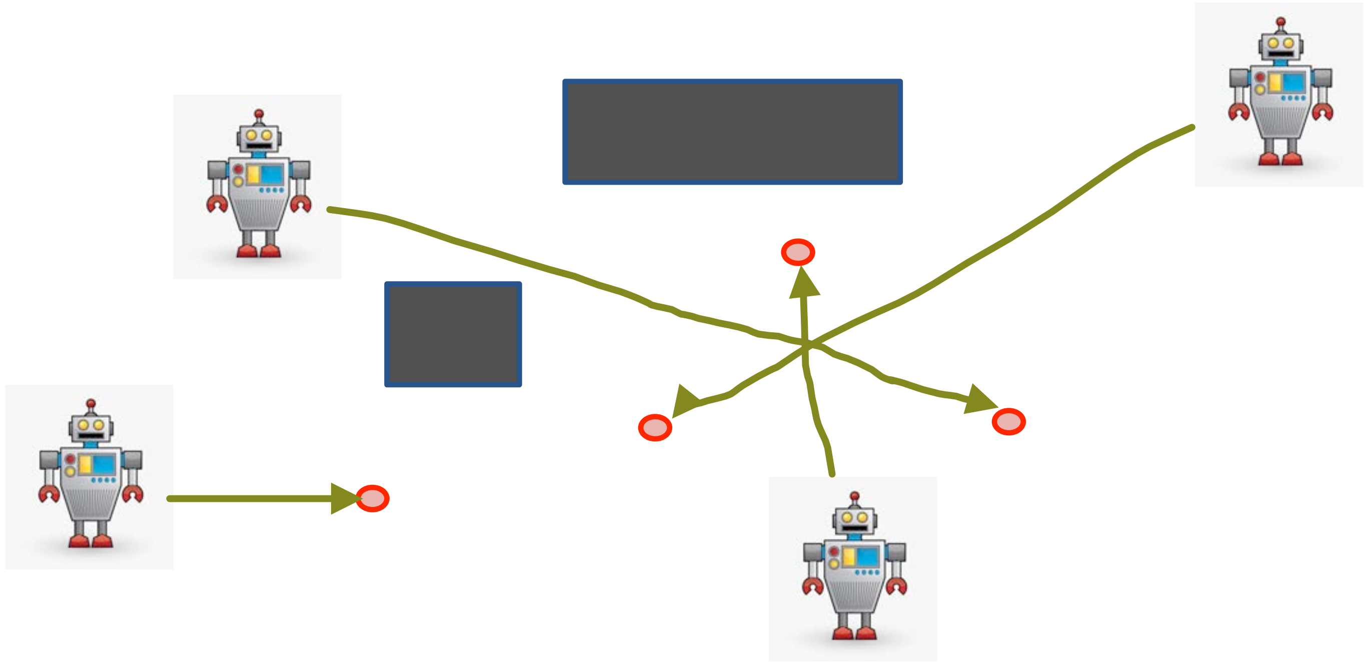


MMP: global planning

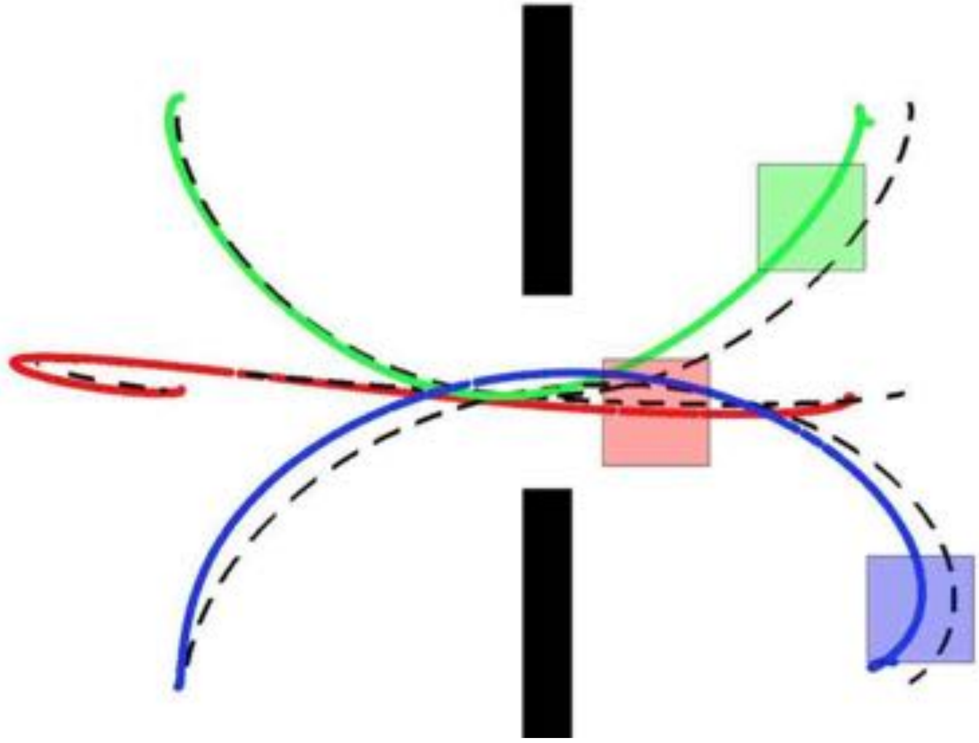
- "Traditional" approaches
  - Assign priorities and sequentially compute trajectories
  - Compute robot paths and adjust velocity profiles

- Optimization-based trajectory generation (examples)
  - "Near"-optimal approaches
    - Continuous space: Mixed Integer Program [Mellinger et al, 2012]
    - Discrete graph: Integer Linear Program [Yu and Rus, 2015]

  - Locally optimal approaches
    - Continuous obstacle-free: SCP [Augugliaro et al, 2012]
    - Continuous with obstacles: SCP [Chen et al, 2015]
    - Continuous 2D: Message passing [Bento et al, 2013]

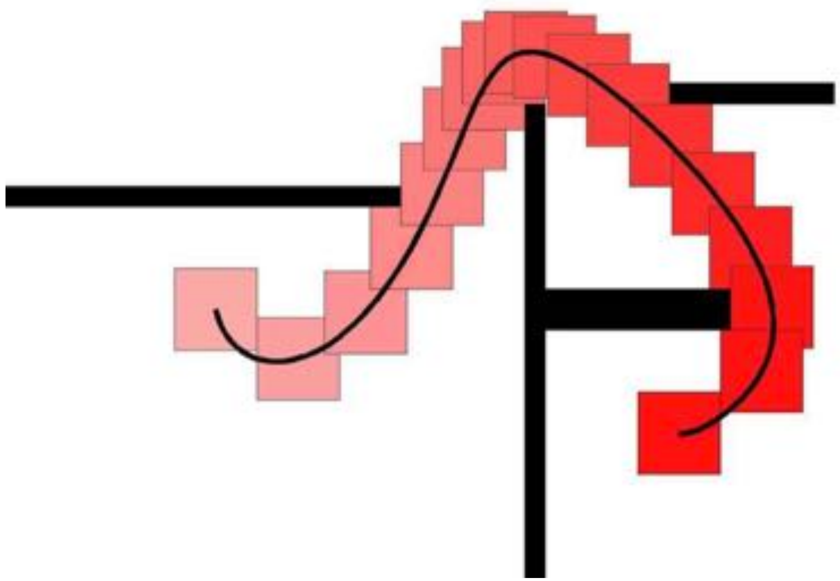
MMP: centralized global planning

- Optimal trajectories, continuous, with dynamics [Mellinger et al, 2012]
- Formulated as a Mixed Integer Program



MMP: centralized global planning

- Optimal trajectories, continuous, with dynamics [Mellinger et al, 2012]
- Formulated as a Mixed Integer Program
  - Trajectory = piecewise polynomial functions over n_w time intervals using Legendre polynomial basis functions $P_{pw}(t)$
  - Minimize the integral of the square of the norm of the snap (the second derivative of acceleration, $k_r = 4$)



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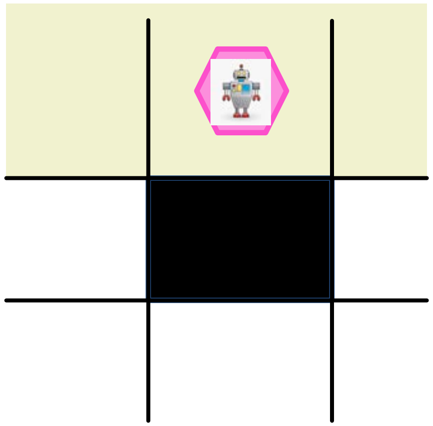
  - Minimize the integral of the square of the norm of the snap (the second derivative of acceleration, $k_r = 4$)
  - Integer constraints for obstacle avoidance
    - At least one of the linear constraints defined by the faces of the obstacle separates the obstacle from the robot volume

$$\mathbf{n}_{of} \cdot \mathbf{r}_T(t_k) \le s_{of} + M b_{ofk} \quad \forall f = 1, ..., n_f(o)$$

$$b_{ofk} = 0 \text{ or } 1 \quad \forall f = 1, ..., n_f(o)$$

$$\sum_{f=1}^{n_f(o)} b_{ofk} \le n_f(o) - 1$$

M>>0



- Optimal, but computationally expensive

MMP: centralized global planning

- Near-optimal planning on a discrete graph [Yu and Rus, 15]
- Formulated as an Integer Linear Program (efficient)

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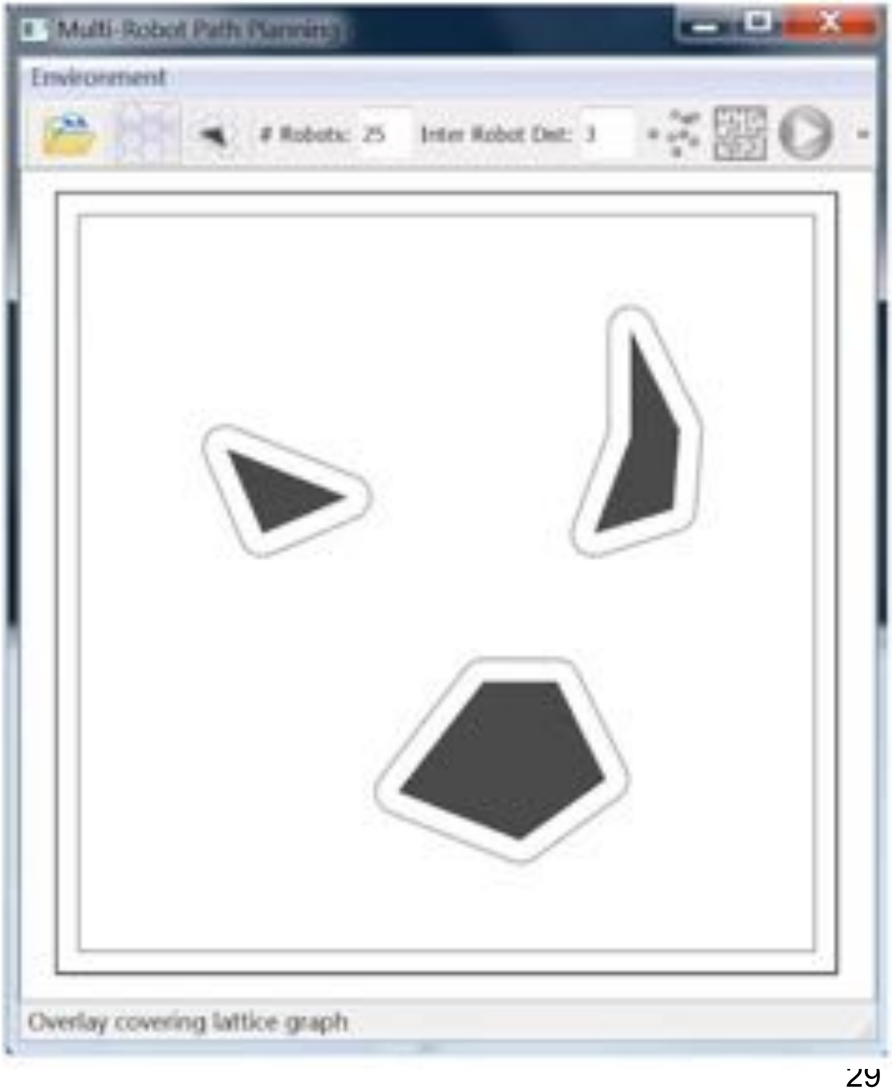
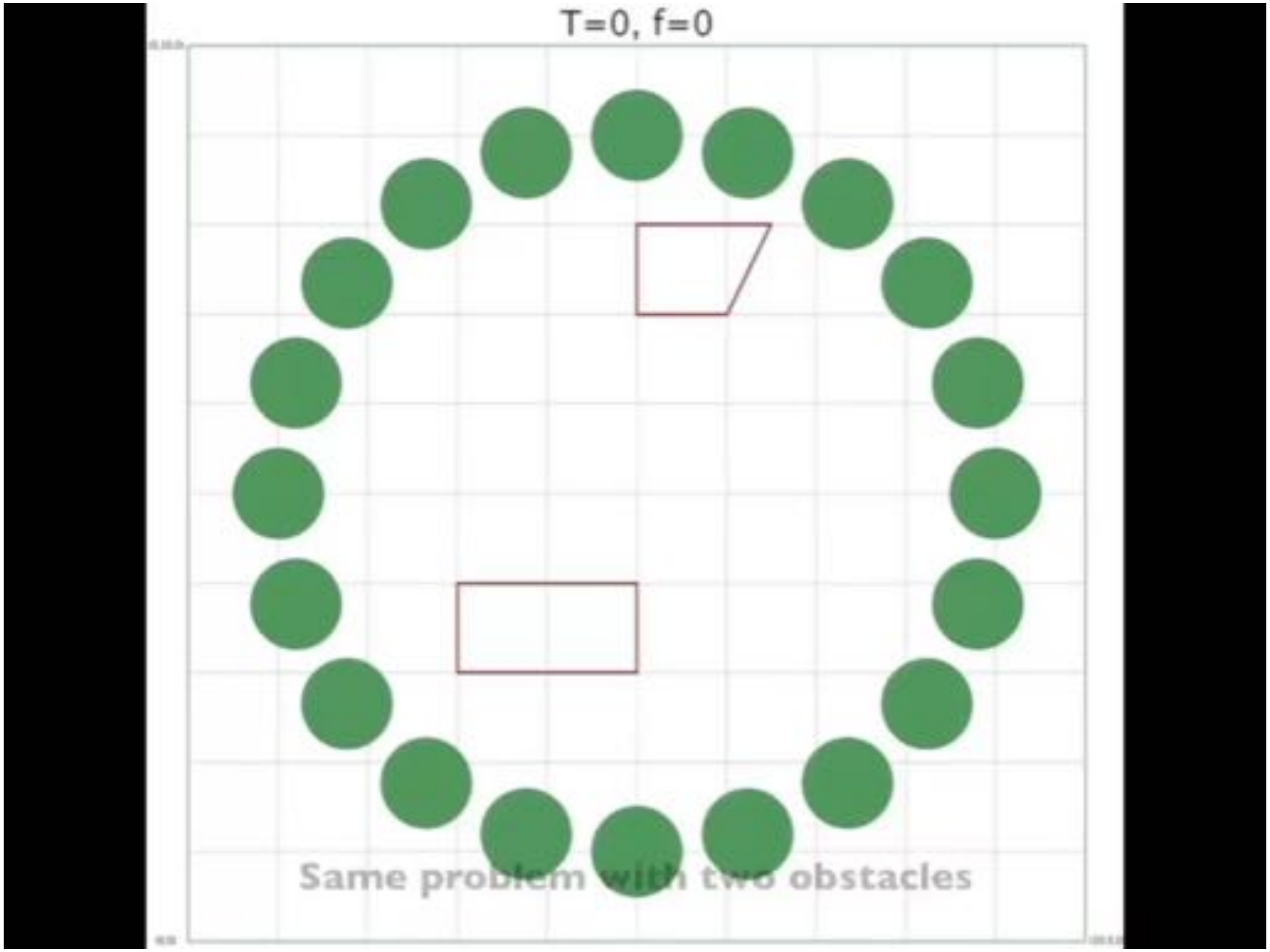


MMP: centralized global planning

- Locally optimal, continuous, 2D, holonomic, parallelizable
- ADMM – 3 weight message passing [J. Bento et al, 2013]



MMP: centralized global planning

- Locally optimal trajectories in free space, with dynamics
- Sequential convex programming (efficient) [Augugliaro et al, 2012]
  - The optimization variable $\chi \in R^{3NK}$ consists of the vehicles' accelerations at each time step k
  - The optimality criterion is the sum of the total thrust at each time step
  - Convex constraints: physical properties of vehicles'
  - Non-convex constraints: collision avoidance:

$$\|p_i[k] - p_j[k]\|_2 \geq R, \quad \forall i, j, \quad i \neq j, \quad \forall k$$

  - Linearized around the current solution results in QP:

$$\begin{aligned} \text{minimize} \quad & \chi^T P \chi + q^T \chi + r \\ \text{subject to} \quad & A_{eq}\chi = b_{eq} \\ & A_{in}\chi \preceq b_{in}, \end{aligned}$$

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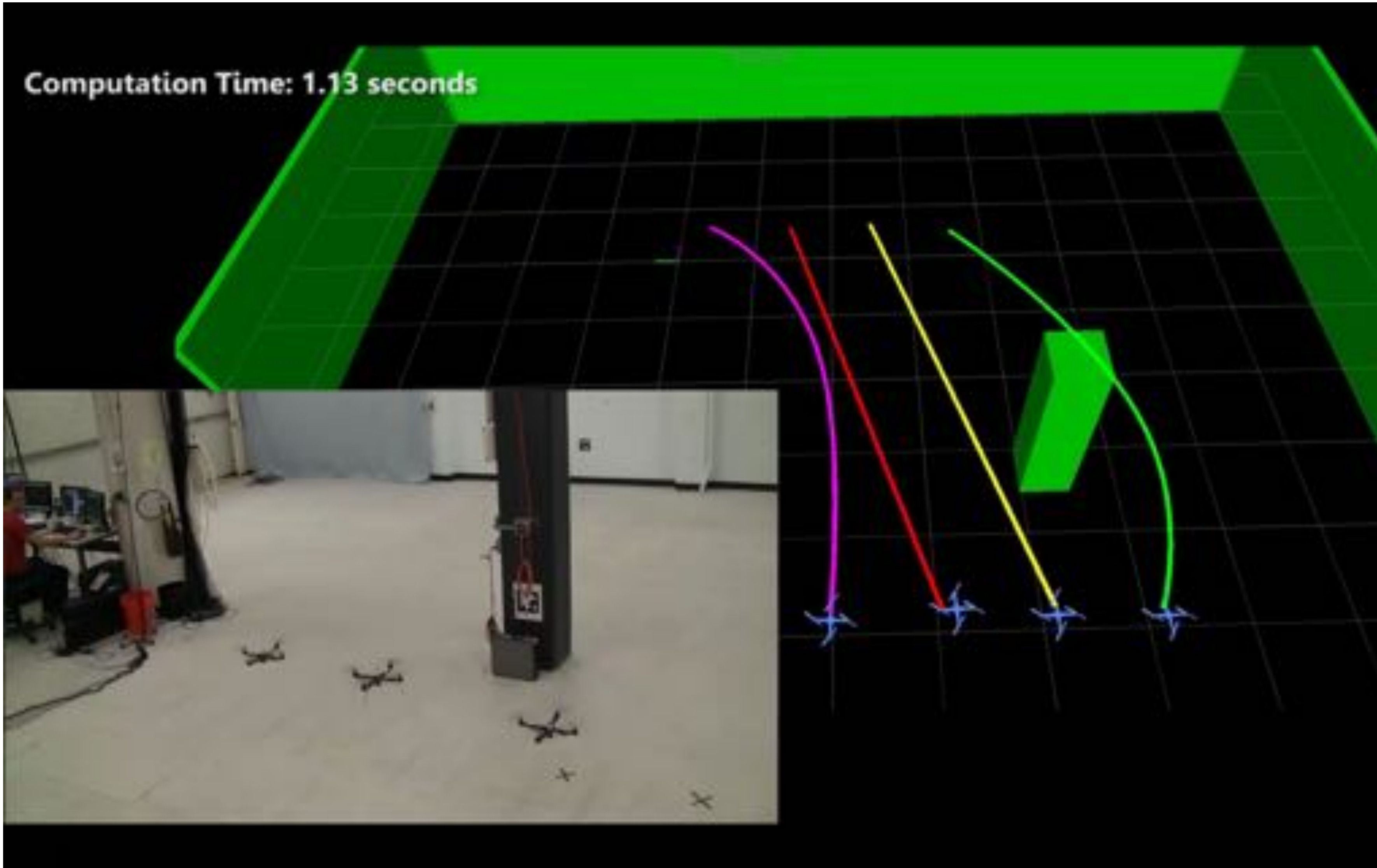
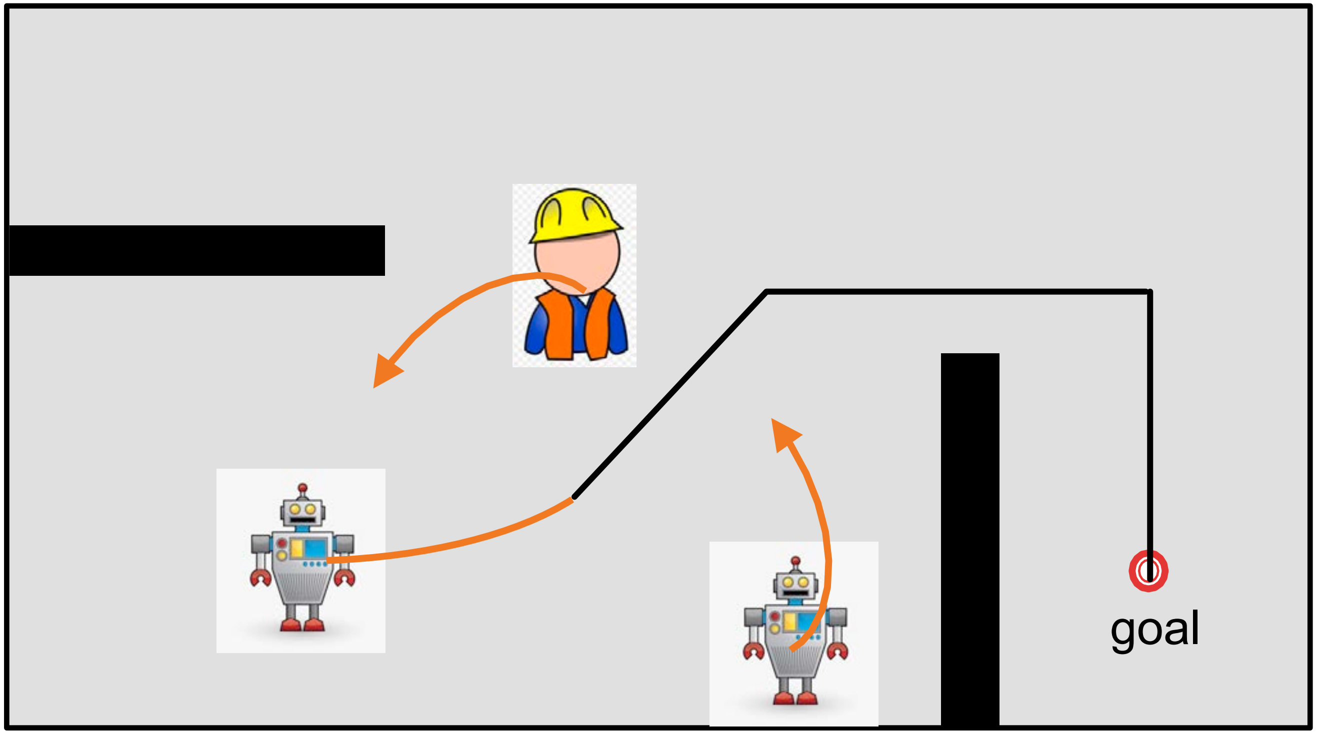


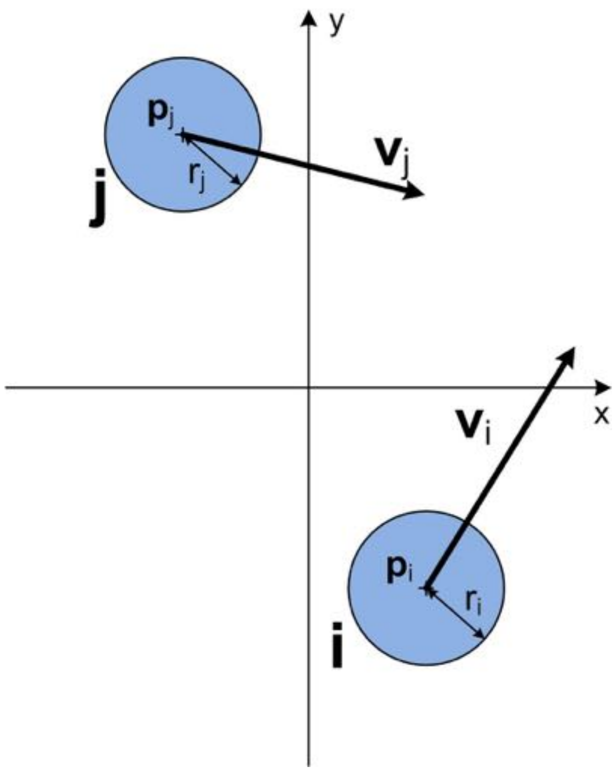
Multi-robot motion planning: problem definition

- Global planning
  - Trajectory from initial to final state
- **Local planning (collision avoidance)**
  - **Trajectory from initial state up to a short time horizon**



MMP: collision avoidance

- Velocity obstacles with motion constraints [Alonso-Mora et al. 2010]
  - Set of motion primitives towards linear trajectories (reference velocity)
  - Collision avoidance constraints in reference velocity space



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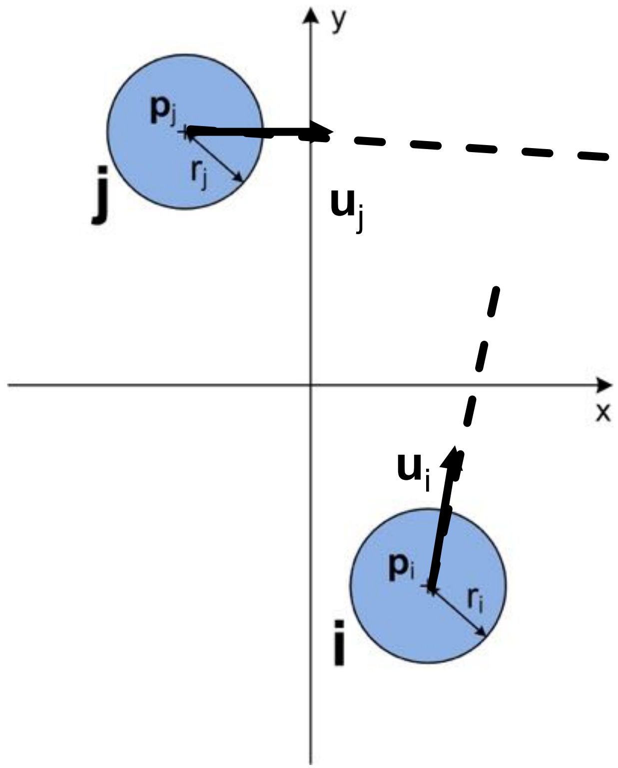

$$||(\mathbf{p}_i + \mathbf{u}_i t) - (\mathbf{p}_j + \mathbf{u}_j t)|| > r_i + r_j$$

$$\forall t \in [0, \tau]$$

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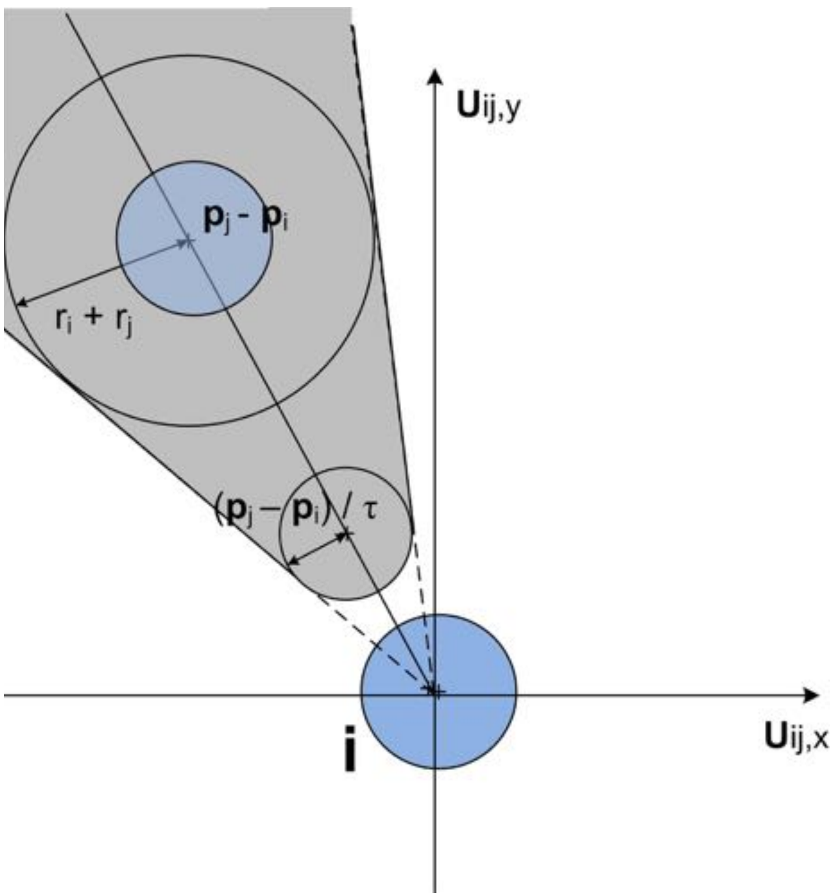

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$$\forall t \in [0, \tau]$$

$$||\frac{\mathbf{p}_i - \mathbf{p}_j}{t} + \underline{(\mathbf{u}_i - \mathbf{u}_j)}|| > \frac{r_i + r_j}{t}$$

Distributed with assumption on $\mathbf{u}_j$

- Static: $\mathbf{u}_j = 0$
- Constant velocity: $\mathbf{u}_j = \mathbf{v}_j$
- Both decision-making:
  - Collaborative

    $\Delta \mathbf{v}_i = \lambda \Delta \mathbf{v}_{ij}$

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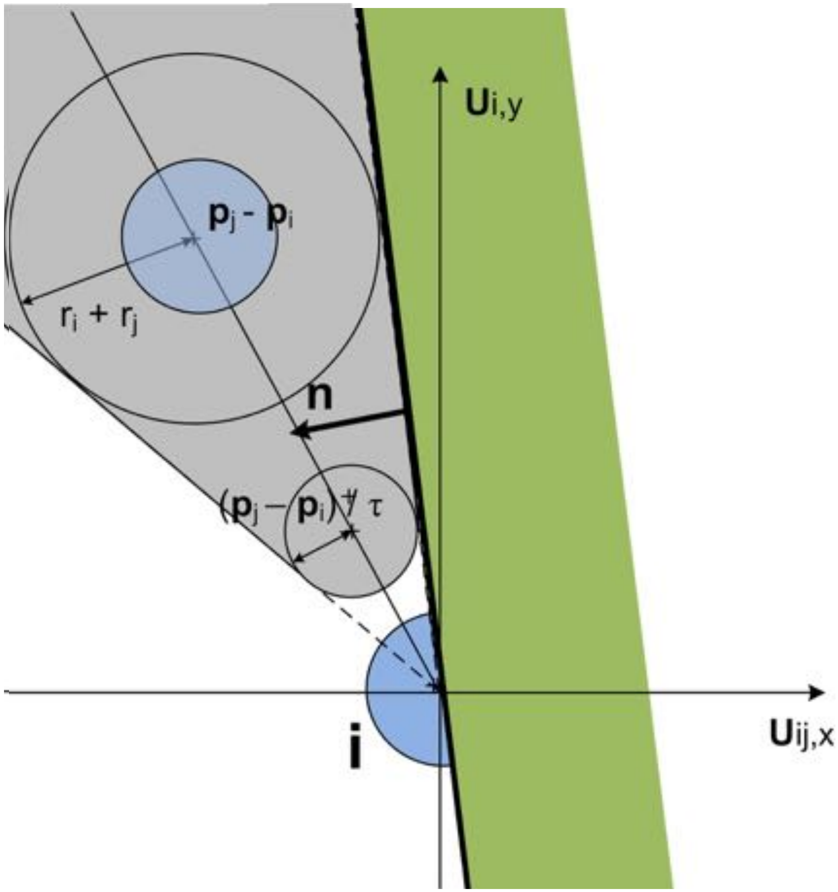

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- Both decision-making:
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    $\Delta \mathbf{v}_i = \lambda \Delta \mathbf{v}_{ij}$

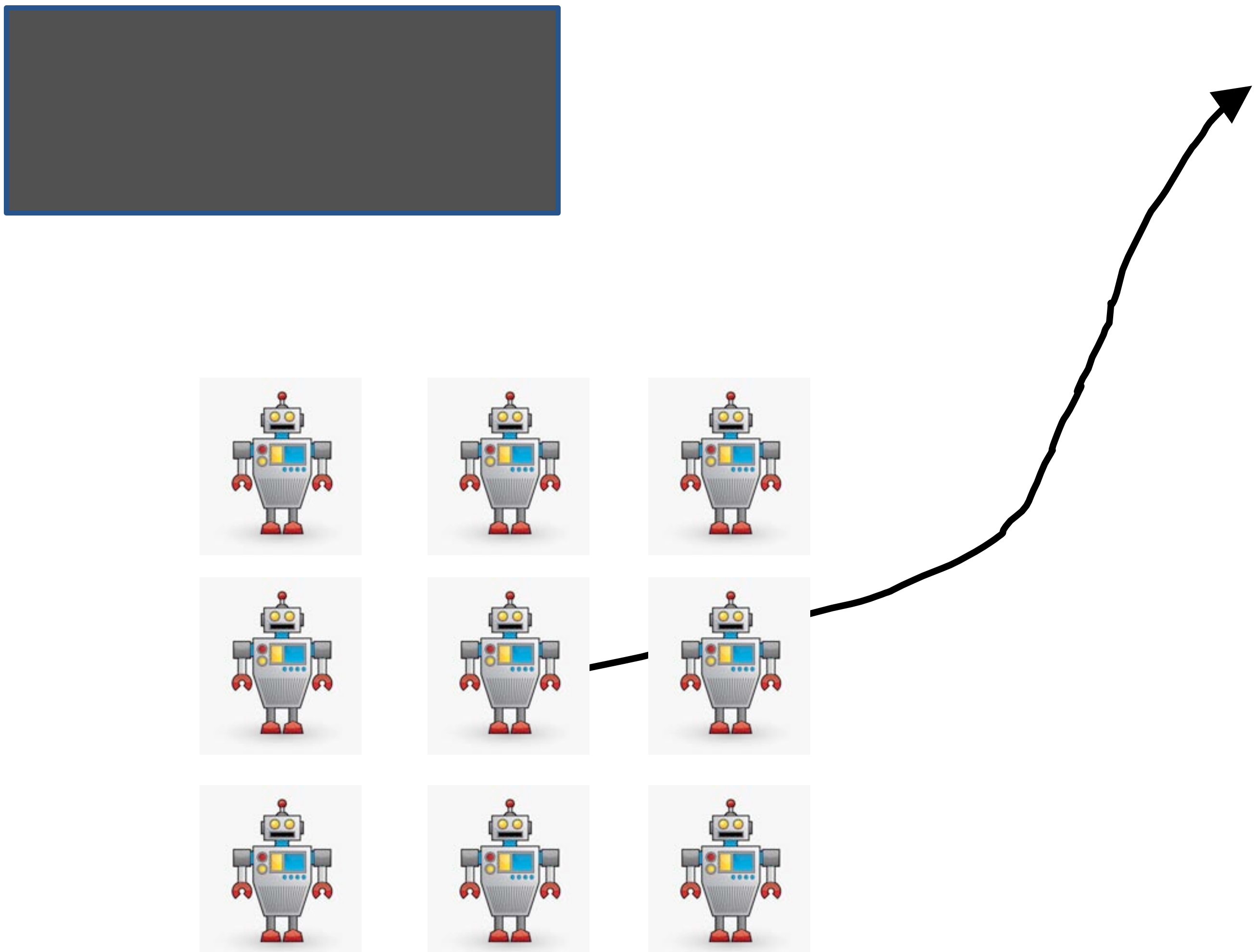
- This gives a distributed convex optimization with linear constraints

MMP: collision avoidance

- Optimal control [Hoffmann and Tomlin 2008]
- Model predictive control [Shim, Kim and Sastry 2003]
- Convex optimization in velocity space [van den Berg et al. 2009]
  - Extension to account for robot dynamics [Alonso-Mora et al. 2010]
  - Also applied to aerial vehicles [Alonso-Mora et al. 2015]

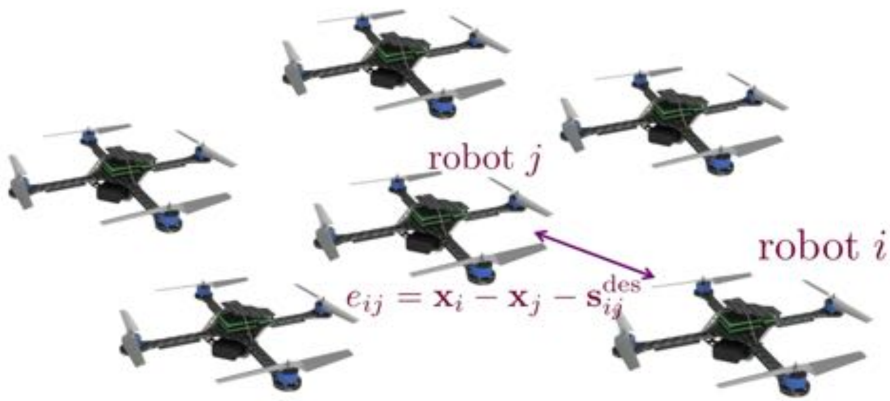
Formation control/planning: problem definition

- Maintain desired inter-robot distances defining the formation

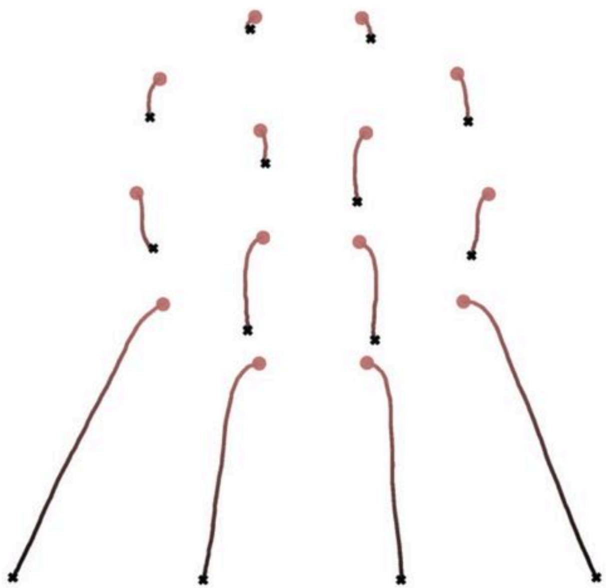


Formation control

- Obstacle-free environments
  - Centralized optimal coverage with assignment [Alonso-Mora et al. 2012]
  - Leader follower with optimal control [Ji, Muhammad and Egerstedt 2006]
  - Distributed QP with leader follower [Turpin, Michael and Kumar 2012]
  - Model Predictive Control [Dunbar and Murray 2002]
  - Distributed consensus [Montijano and Mosteo 2014]



[Turpin, Michael and Kumar 2011]



[Alonso-Mora et al, 2012]

Formation planning: with obstacles

- Convex optimization
  - SDP, circular formation, triangulate space [Derenick and Spletzer 2007]
  - SDP for circular obstacles [Derenick, Spletzer and Kumar 2010]
  - Centralized LP in velocity space [Karamouzas and Guy 2015]
  - Distributed QP in velocity space [Alonso-Mora et al. 2015]
    - Constraints: Avoidance + min/max inter-robot distance

Formation planning: with obstacles

- Distributed convex optimization [Alonso-Mora et al. 2015]
  - Compute a new velocity

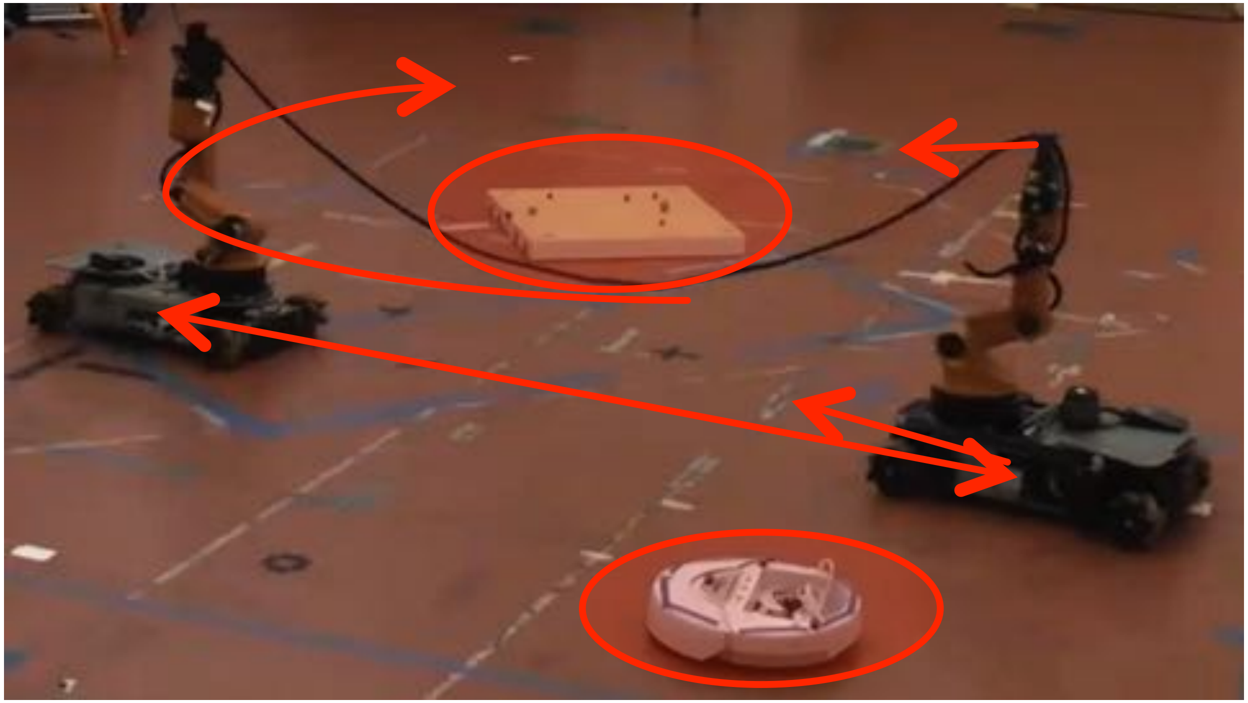
    minimize (deviation to target global motion of the object)

    s.t. Collision avoidance constraints [velocity obstacles]

    Shape maintenance constraints: min / max distance

    Force sensing used to indicate intention and to coordinate

    Constraints convexified & partitioned assuming cooperation

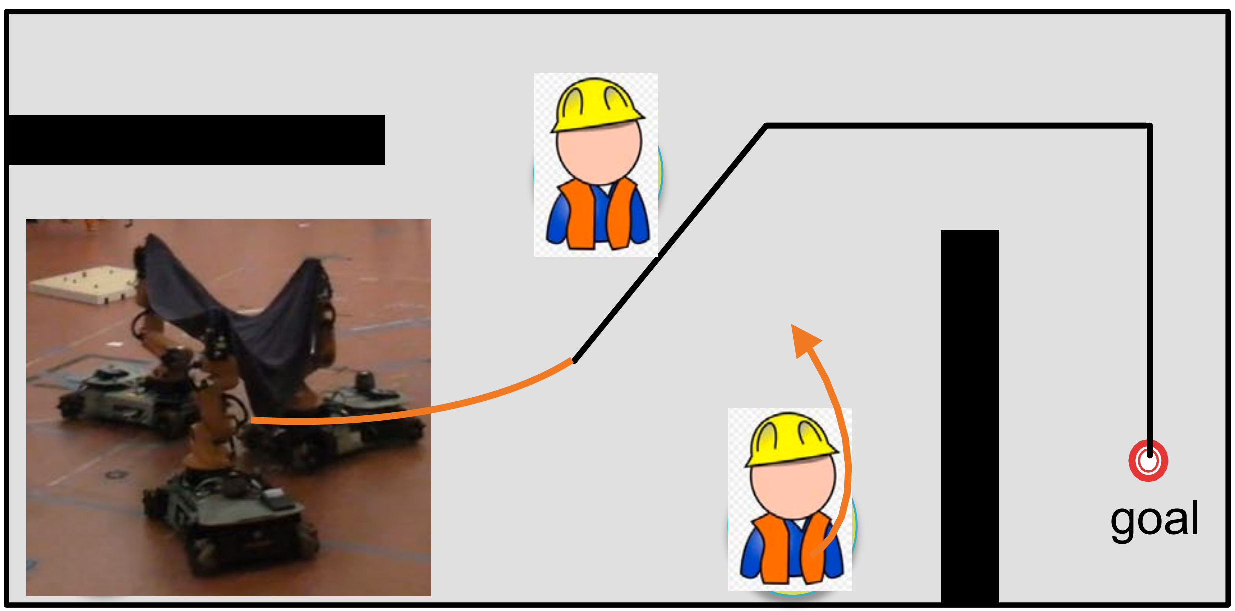


Formation planning

- Distributed convex optimization [Alonso-Mora et al. 2015]

Formation planning

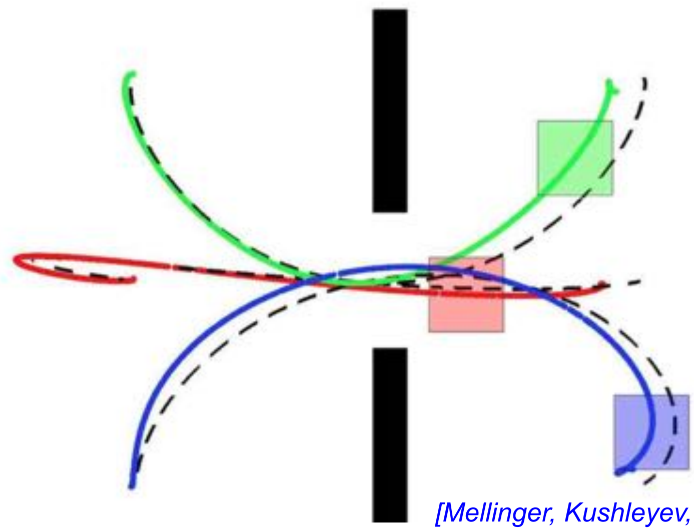
- Convex optimization
  - SOP, circular formation, triangulate space [Derenick and Spletzer 2007]
  - SDP for circular obstacles [Derenick, Spletzer and Kumar 2010]
  - Centralized LP in velocity space [Karamouzas and Guy 2015]
  - Distributed QP in velocity space [Alonso-Mora et al. 2015]
- Non-convex optimization
  - Off-line global MIP for sub-groups [Kushleyev, Mellinger and Kumar 2012]
  - On-line local sequential convex programming [Alonso-Mora et al. 2015]



Formation planning

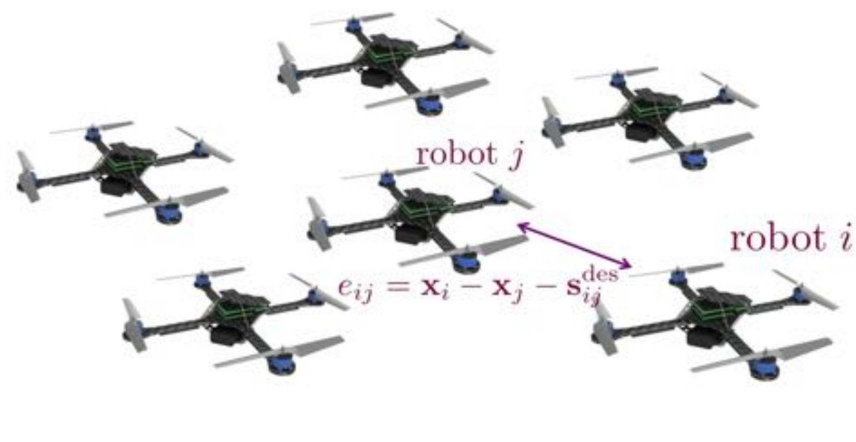
- Centralized off-line MIP subgroups [Kushleyev, Mellinger and Kumar 2012]

MIQP trajectory planning for subgroups of fixed formation



[Mellinger, Kushleyev, Kumar, 2012]

Distributed formation control within the subgroup



[Turpin, Michael and Kumar 2011]

Formation planning

- Centralized off-line MIP subgroups [Kushleyev, Mellinger and Kumar 2012]

Formation planning

- Centralized local real-time SCP [Alonso-Mora et al. 2015]

Goal – Target motion

↓

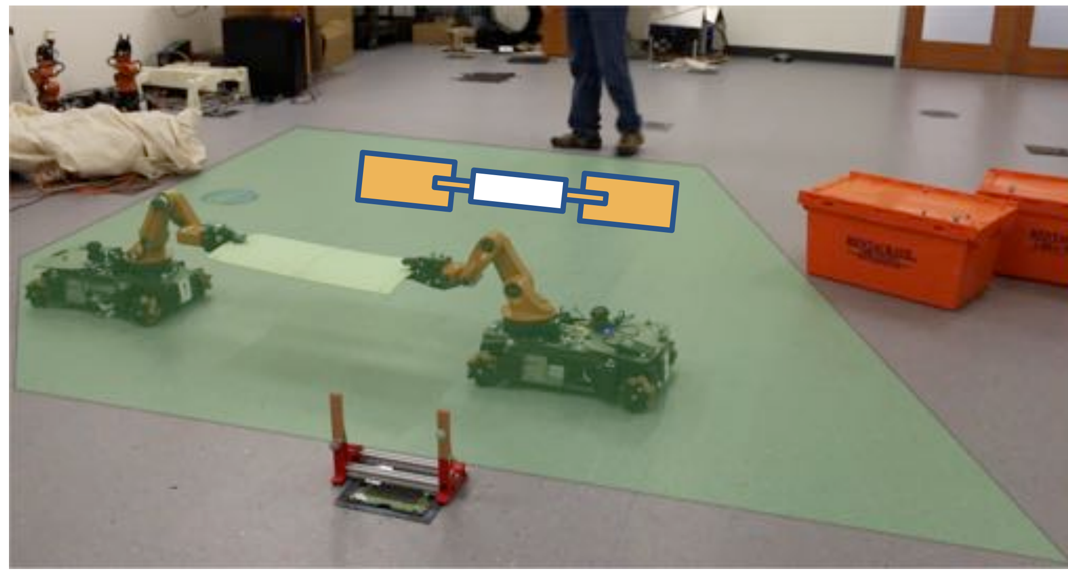
Compute largest convex obstacle-free area
Iterative QP + SDP

↓

Compute optimal formation parameters via a nonlinear constrained optimization SCP

↓

Optimal robot commands

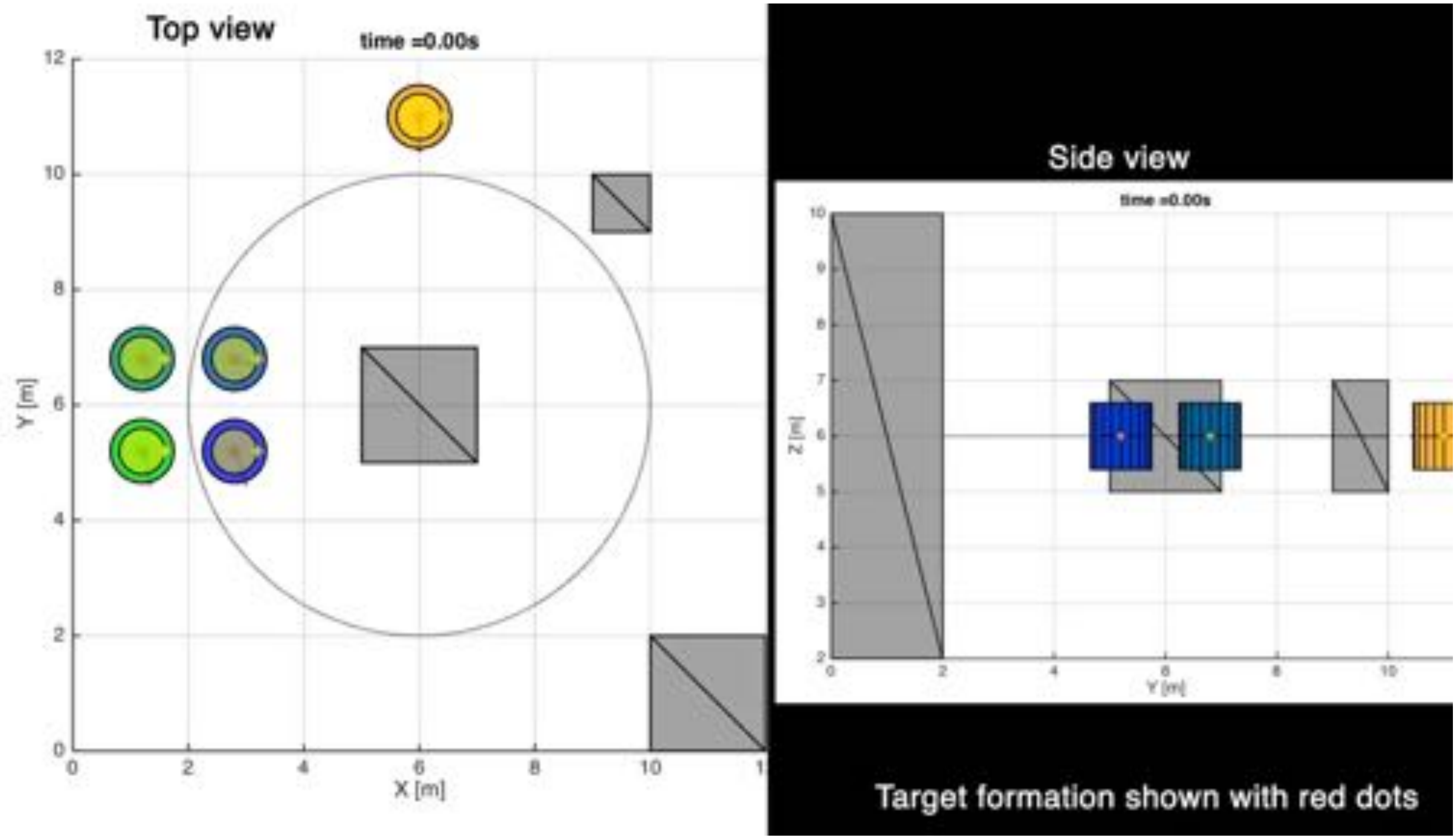


$$\mathbf{x}_i^* = w_t||\mathbf{t}-\mathbf{g}(t_f)||^2+w_s||s-\bar{s}||^2+w_q||\mathbf{q}-\bar{\mathbf{q}}||^2+c_i$$

s.t.	$C_1^j = \{A(\mathbf{t} + s\,\mathrm{rot}(\mathbf{q}, \mathbf{f}_{0,j}^i)) \le \mathbf{b}\}$	Inside convex polytope
	$C_2 = \{s\ d_0^i \ge 2\max(r,h)\}$	Minimum size
	$C_3 = \{\|\|\mathbf{q}\|\|^2 = 1\}$	Quaternion

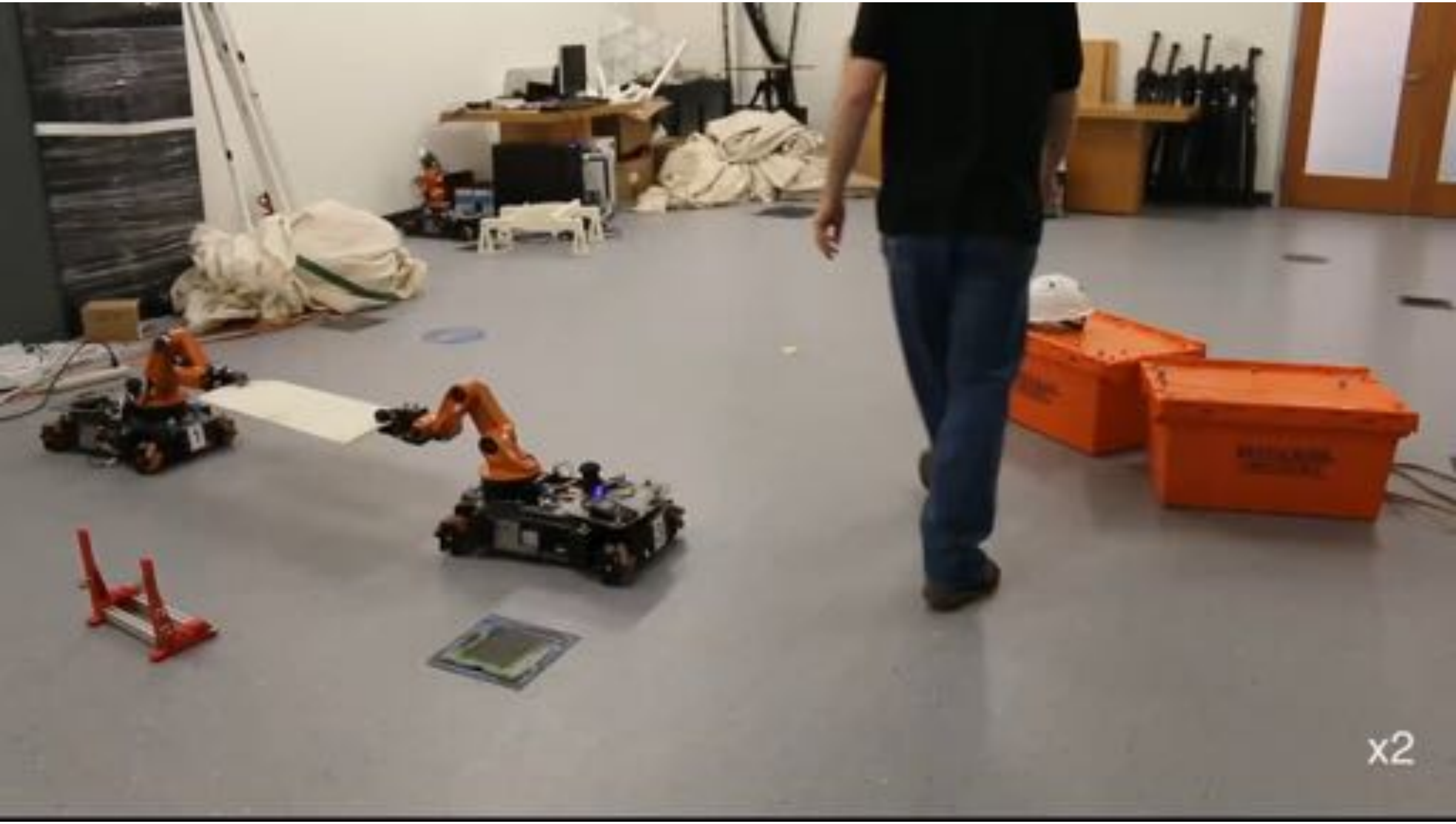
Formation planning

- Centralized local real-time SCP [Alonso-Mora et al. 2015]



Formation planning

- Centralized local real-time SCP [Alonso-Mora et al. 2015]



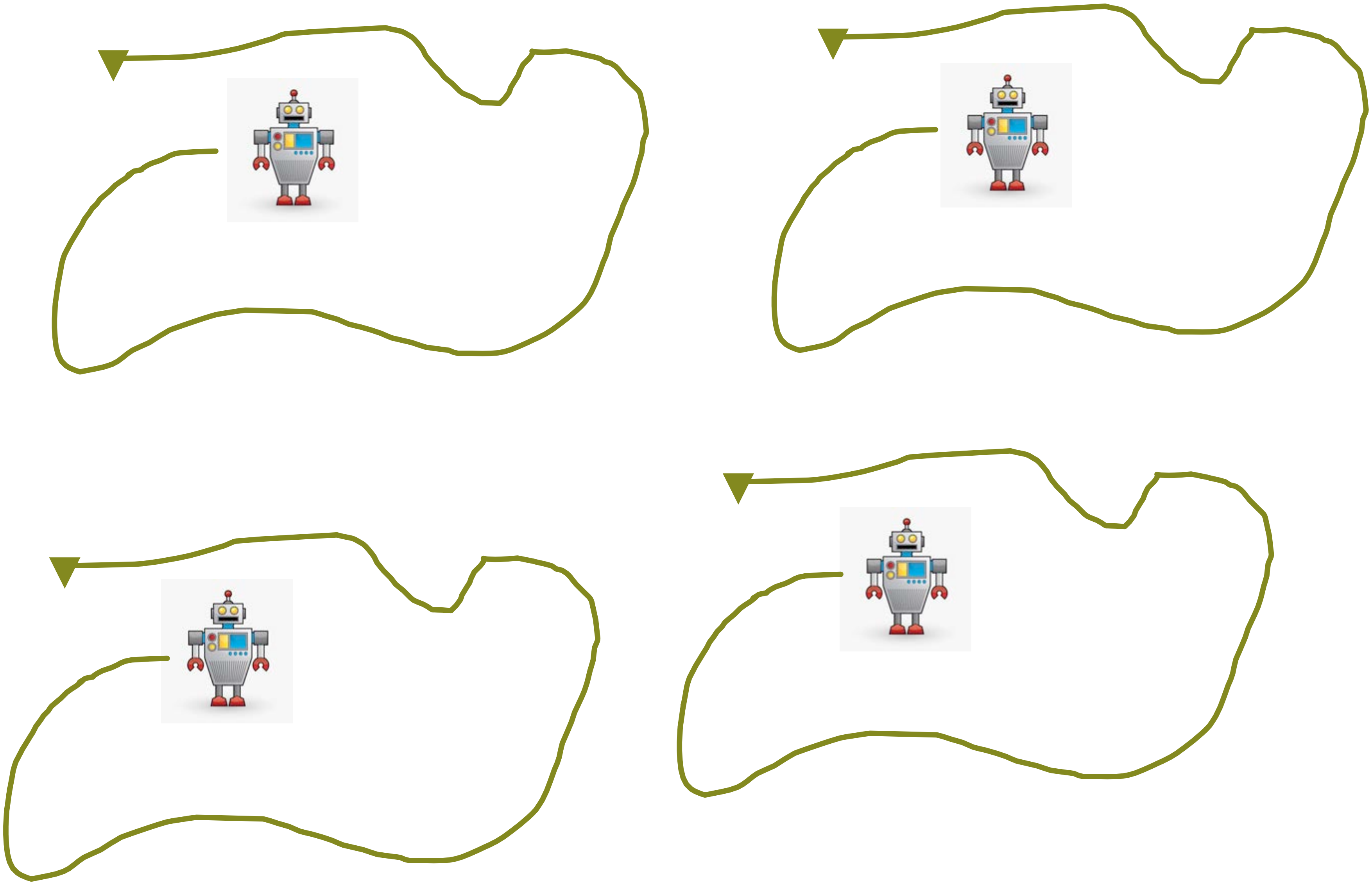
Take home message

- ***Convex*** *optimization with* ***continuous*** *variables* $\boldsymbol{x} \in \mathbb{R}^{\nu}$
  - LP/QP/SDP
    - Very fast, global optimum
    - But, most problems are not convex

- ***Non-convex*** *optimization with* ***continuous*** *variables*
  - *Sequential convex programming* ***SCP*** *[local]*
    - *Fast but local, often works well, but no strict guarantees*

- ***Non-convex*** *optimization with* ***binary*** *variables* $x_j \in \{0, 1\}$
  - *Mixed Integer Program* ***MIP*** *[global]*
    - *Slow but eventually will find the global optimum*

Surveillance and monitoring: problem definition

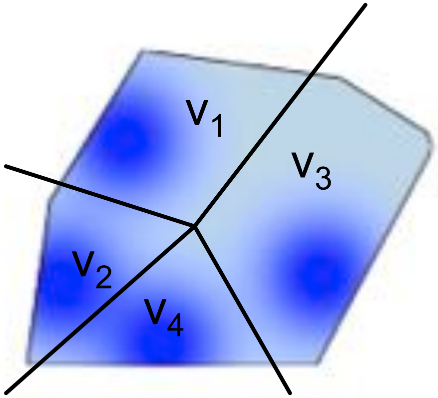


Surveillance and monitoring: problem definition

- Consider m robots at $p = \{p_1,...,p_m\}$
- Environment is partitioned into $v = \{v_1, . . . , v_m\}$
- Cost:

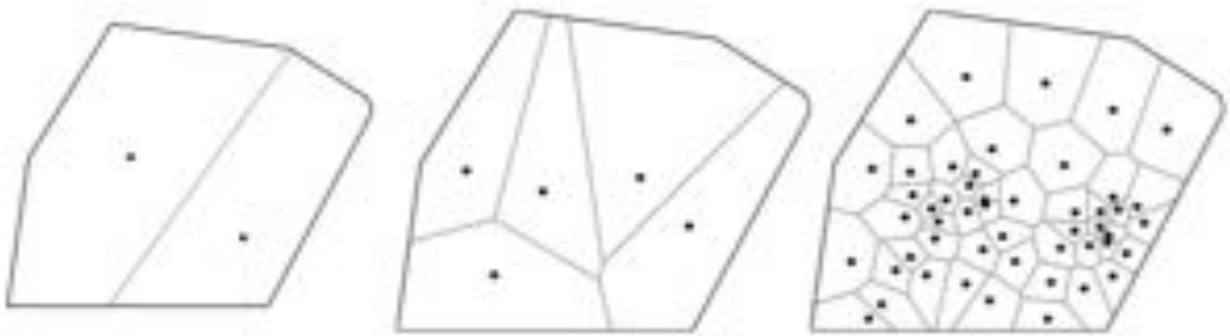
$$\mathcal{H}(p, v) = \sum_{i=1}^{m} \int_{v_i} f(\|x - p_i\|)\varphi(x)dx$$

  - $\varphi : \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ density
  - $f : \mathbb{R}_{\geq 0} \to \mathbb{R}$ penalty function



- Voronoi partition $\{V_1, . . . , V_m\}$ generated by points $\{p_1, . . . , p_m\}$

$$V_i(p) = \{x \in \mathcal{Q} | \ \|x - p_i\| \leq \|x - p_j\| , \forall j \neq i\}$$



Descartes 1644, Dirichlet 1850, Voronoi 1908, Thiessen 1911, Fortune 1986 (sweepline algorithm $O(m \log(m))$)

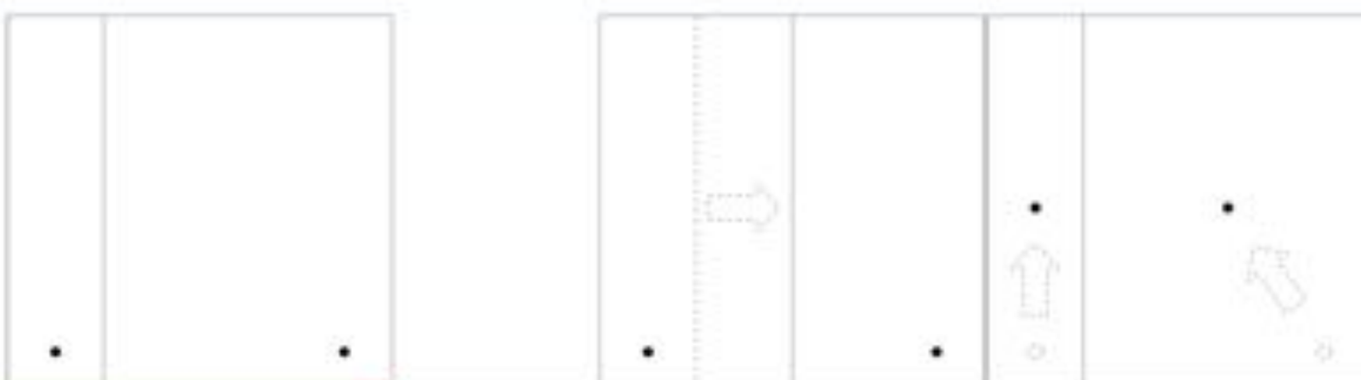
Surveillance and monitoring: problem definition

$$\mathcal{H}(p, v) = \sum_{i=1}^{m} \int_{v_i} f(\|x - p_i\|)\varphi(x)dx$$

Theorem (Alternating Algorithm, Lloyd '57)

1. *at fixed positions, optimal partition is Voronoi*
2. *at fixed partition, optimal positions are "generalized centers"*
3. *alternate v-p optimization*

$\implies$ *local optimum = center Voronoi partition*



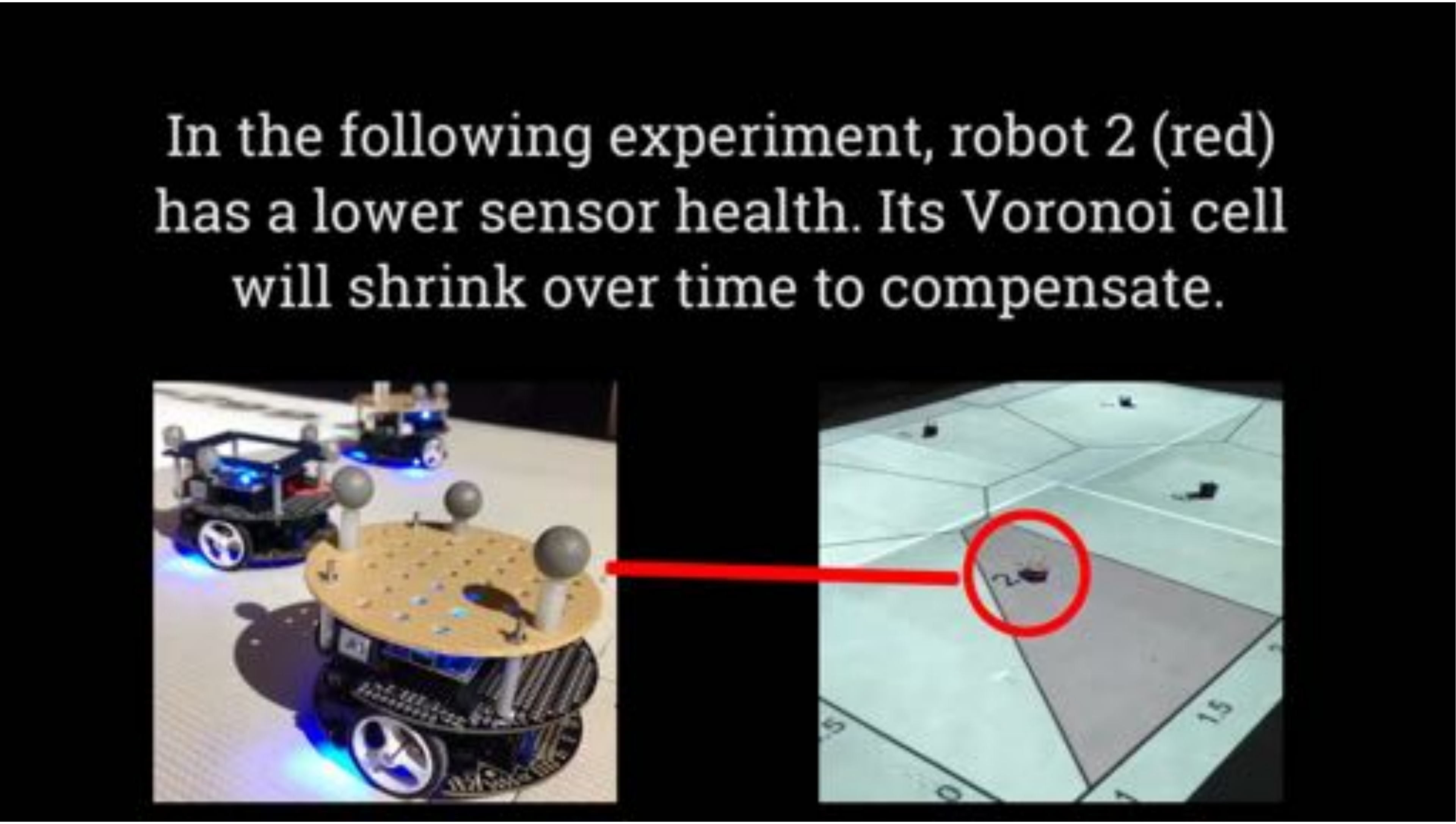
Surveillance and monitoring

- Spatial distribution known
  - Gradient descent – alternating algorithm [Lloyd 1982]

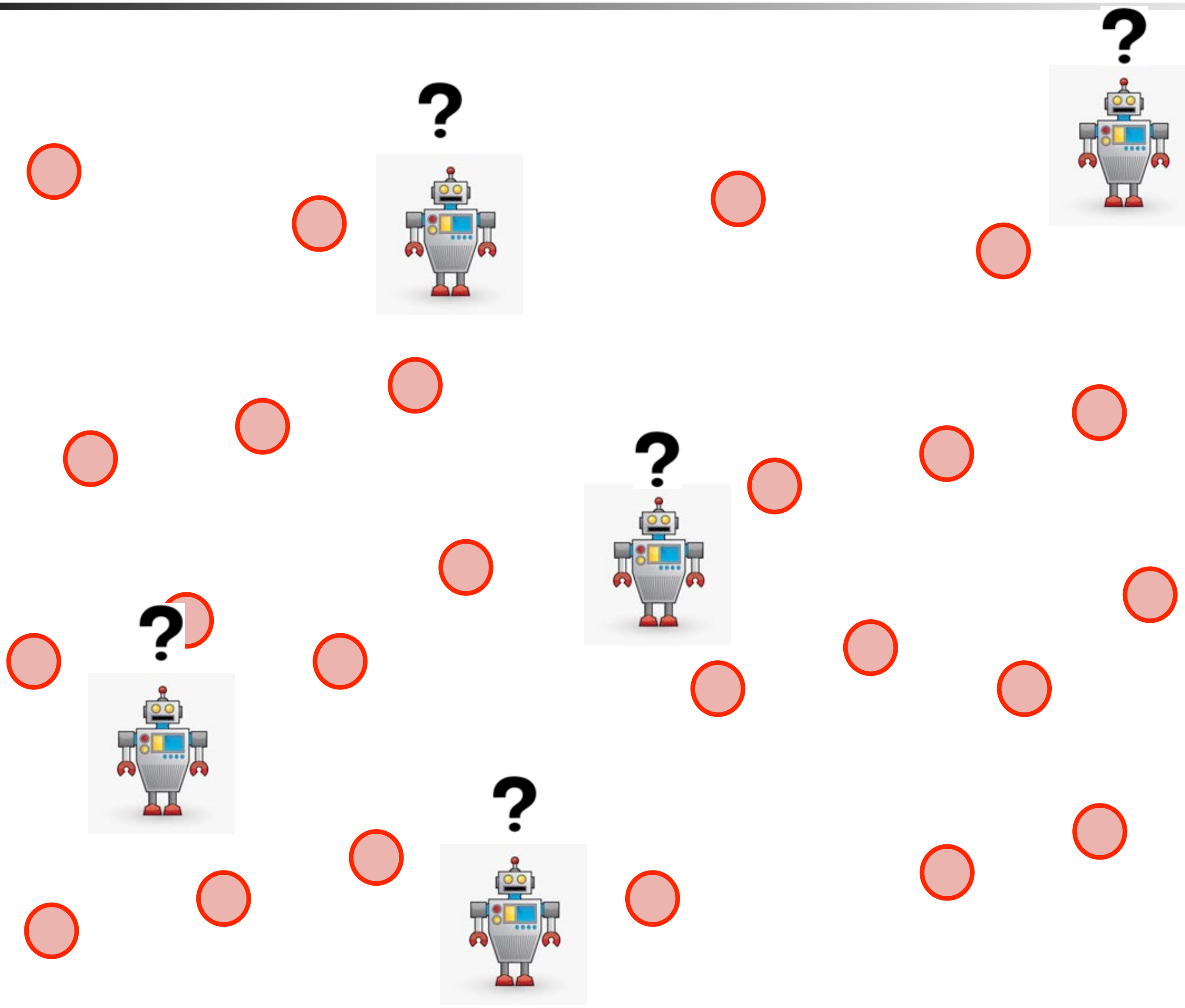
- Spatial distribution unknown
  - Adaptive algorithms [Schwager, Rus and Slotine 2009]
  - Motion constraints [Savla and Frazzoli 2010]
  - Persistent surveillance [Smith et al. 2011]
  - Adapting to sensing/actuation [Pierson et al. 2015]

Multi-robot coverage

- Adapting to sensing/actuation [Pierson et al. 2015]



Task assignment: problem definition



- Taxonomy [Gerkey 2004]

Task assignment: Single task per robot

3. THE GENERAL ASSIGNMENT PROBLEM

Suppose n individuals ($i = 1, \ldots, n$) are available for n jobs ($j = 1, \ldots, n$) and that a rating matrix $R = (r_{ij})$ is given, where the r_{ij} are positive integers, for all i and j. An assignment consists of the choice of one job j_i for each individual i such that no job is assigned to two different men. Thus, all of the jobs are assigned and an assignment is a permutation

$$\begin{pmatrix} 1 & 2 & \ldots & n \\ j_1 & j_2 & \ldots & j_n \end{pmatrix}$$

of the integers 1, 2, ..., n. The General Assignment Problem asks:

For which assignments is the sum

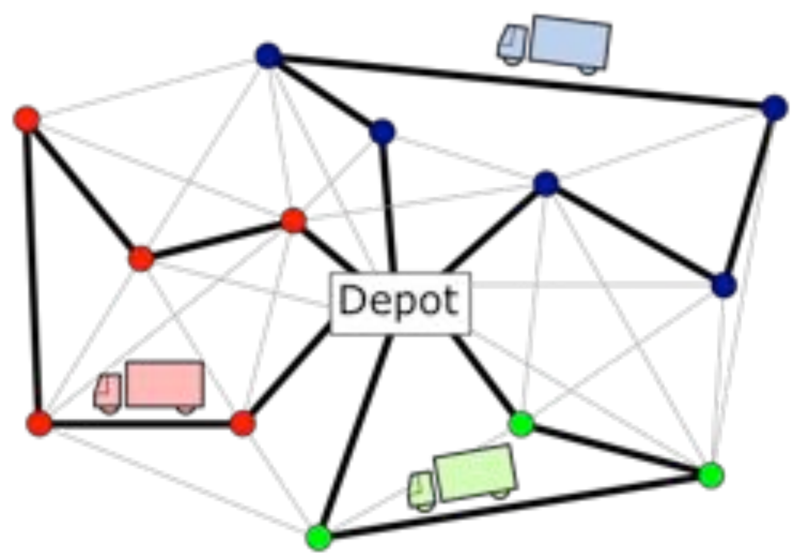
$$r_{1j_1} + r_{2j_2} + \ldots + r_{nj_n}$$

of the ratings largest?

- Optimal [Kuhn 1955]
- Suboptimal: auction [Bertsekas 1992]
- Concurrent assignment and planning [Turpin, Michael, Kumar 2014]

Task assignment: vehicle routing

- A pot. large number of tasks to be satisfied by a set of robots
- Static vehicle routing [Toth and Vigo 2001]
  - Traveling salesman problem
    - Small problems can be solved via a MIP
    - Large problems are typically solved with heuristics (tabu search)
- Dynamic vehicle routing [Bertsimas and van Ryzin 1991]
  - Introduced queuing theory (Arrival process: spatio-temporal Poisson)



Task assignment: vehicle routing

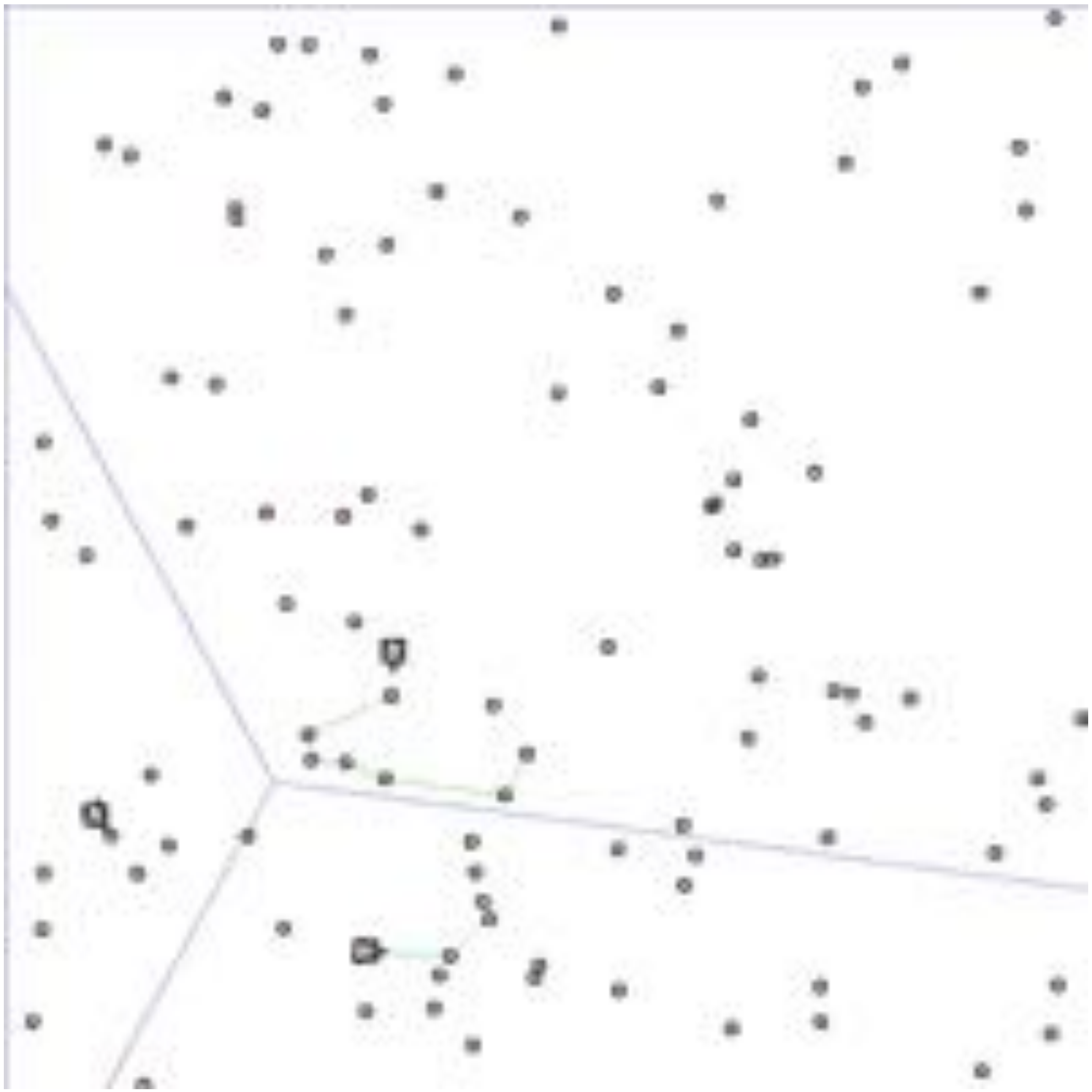
- A potentially large number of tasks are to be satisfied by a set of robots

- Static vehicle routing [Toth and Vigo, 2001]
  - Traveling salesman problem
    - Large problems are typically solved with heuristics (tabu search)

- Dynamic vehicle routing [Bertsimas and van Ryzin, 1991]
  - Introduced queuing theory
  - Motivated many extensions
    - time constraints [Pavone et al, 2009]
    - service priorities [Smith et al, 2009]
    - adaptive and decentralized algorithms [Arsie et al, 2009]
    - complex vehicle dynamics [Savla et al. 2008]
    - limited sensing range [Enright and Frazzoli, 2006]
    - mobility on demand and rebalancing [Smith et al, 2013]

An optimal spatially-unbiased heavy-load policy

- Voronoi partition + single robot TSP [Frazzoli and Bullo, CDC04]



Combination of optimization methods

- Animation display with multiple robots [Alonso-Mora et al. 2012]
  - Optimal coverage, goal assignment and collision avoidance

Combination of optimization methods

- Animation display with multiple robots [Alonso-Mora et al. 2012]
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Overview

- Introduction
- 1. Optimal control and optimization tools
- 2. Problem definition & overview of state of the art
- **Summary**

Summary

- Optimal control / optimization techniques can play an important role in the design and operation of multi-robot systems

- We provided an overview of these techniques in the context of four major classes of multi- robot problems:
  - Multi-robot motion planning
  - Formation planning
  - Task assignment
  - Surveillance & monitoring

- Optimization methods can also be found in other areas, such as cooperative localization and mapping

Questions?

- Optimal control / optimization techniques can play an important role in the design and operation of multi-robot systems

- Please send me more refs. and we will add them!
- **Contact: J. Alonso-Mora: jalonsom@mit.edu**

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